International Journal of Theoretical Physics, Vol. 40, No. 5, 2001

Gravitational Decoherence and EPR Correlations

Z. Haba[1]

Received May 22, 2000

It is shown that gravitons in a mixed (thermal) state can lead to a decoherence in large quantum systems. As a consequence the nonlocal Einstein–Podolsky–Rosen phenomena resulting from quantum coherence can disappear in some particle states after a quantization of Einstein gravity.

1. INTRODUCTION

Quantum coherence leads to remarkable phenomena of an immediate change of a state of a distant subsystem entangled with another subsystem undergoing a measurement (the Einstein–Podolsky–Rosen paradox (Einstein *et al.*, 1935), EPR for short). In general, if the system is not closed, an effect of measurement on a small part of it will spread over all constituents. As a consequence, it can be negligibly small for large distances. The influence of an environment on the coherence has been discussed in Zurek (1982) and Stern *et al.* (1990). In this paper we consider an environment of quantized gravitational waves (decoherence resulting from the quantum gravity at zero temperature has been discussed also in Ellis *et al.* (1989) and Anastopoulos (1996)). Then, we discuss the EPR experiment of measuring a momentum of one of two particles in an entangled state. It is shown that owing to the disappearance of the coherence the momentum of the distant second particle remains undetermined. One could interprete the effect of an indeterminate momentum either as a description of quantum system by dissipative dynamics or as a result of scattering by gravitons in a Hamiltonian system. The effect is small for elementary particles. Then, we would have to wait for a long time before the coherence is lost. However, the decoherence rate for a cluster of N particles increases at least linearly with N. Hence, it becomes efficient for macroscopic bodies.

[1] Institute of Theoretical Physics, University of Wroclaw, Wroclaw, Poland; e-mail: zhab@ift.uni.wroc.pl

0020-7748/01/0500-0985$19.50/0

We consider a semiclassical (complex) solution of the Klein–Gordon equation

$$\Box_g \psi = M^2 c^2 \psi \tag{1}$$

where

$$\Box_g = g^{\mu\nu}\partial_\mu\partial_\nu + g^{\alpha\beta}\Gamma^{\nu}_{\alpha\beta}\partial_\nu$$

$g^{\mu\nu}$ is the Riemannian metric and $\Gamma^{\nu}_{\alpha\beta}$ are the Christoffel symbols.

In the semiclassical approximation we write

$$\psi = \exp\left(\frac{i}{\hbar}W\right)\phi \tag{2}$$

Then, Eq. (1) in the leading order of $\hbar$ gives equations for W and ϕ

$$-g^{\mu\nu}\partial_\mu W \partial_\nu W = M^2 c^2 \tag{3}$$

and

$$g^{\mu\nu}\partial_\mu W \partial_\nu \phi = 0 \tag{4}$$

We consider a perturbation of the metric around the flat one $\eta^{\mu\nu} = (1, 1, 1, -1)$

$$g^{\mu\nu} = \eta^{\mu\nu} + 2\alpha^{\mu\nu} \tag{5}$$

We solve Eq. (3), the Hamilton–Jacobi equation, perturbatively in α

$$W = W^{(0)} + W^{(1)} \tag{6}$$

where the zeroth order $\exp\left(\frac{i}{\hbar}W^{(0)}\right)$ describes the plane wave, that is,

$$W^{(0)} = Px \tag{7}$$

with $P^2 = -M^2c^2$. The equation for $W^{(1)}$ reads

$$\alpha^{\mu\nu}\partial_\mu W^{(0)}\partial_\nu W^{(0)} + \eta^{\mu\nu}\partial_\mu W^{(0)}\partial_\nu W^{(1)} = 0 \tag{8}$$

We choose the transverse-traceless gauge for α. In such a case α has only spatial components. Then, the solution of Eq. (8) is

$$W^{(1)}(x_0, \mathbf{x}) = P_0^{-1}\int_0^{x_0} \alpha^{jl}(x_0 - \tau, \mathbf{x} + \tau\mathbf{P}/P_0)P_j P_l \, d\tau \tag{9}$$

Eq. (4) can be considered as a transport equation in x_0. Its solution is determined by the solution $\mathbf{q}(x_0, \mathbf{x})$ of the equation

$$\frac{dq^j}{dx_0} = g^{jk}\partial_k W (g^{0\mu}\partial_\mu W)^{-1} \tag{10}$$

In the zeroth order in α (which will be sufficient for our purposes)

$$\mathbf{q}(x_0, \mathbf{x}) = \mathbf{x} + x_0\mathbf{P}/P_0 \tag{11}$$

2. QUANTIZATION OF GRAVITATIONAL WAVES

We quantize the gravitational waves expanding α in the momentum space

$$\alpha_a^b(x) = \sqrt{4\pi c}\sqrt{\hbar\kappa/c^4}(2\pi)^{-3/2}\int d\mathbf{k}\,|\mathbf{k}|^{-1/2}\exp(i\mathbf{kx})\sum_{\zeta=1,2}\big(\varepsilon_a^b(\zeta,k)C(\zeta,\mathbf{k})$$
$$\times\exp(-i|\mathbf{k}|x_0) + \overline{\varepsilon_a^b}(\zeta,k)C(\zeta,\mathbf{k})^+\exp(i|\mathbf{k}|x_0)\big) \tag{12}$$

where

$$[C(\zeta,\mathbf{k}), C(\zeta',\mathbf{k}')^+] = \delta_{\zeta\zeta'}\delta(\mathbf{k}-\mathbf{k}') \tag{13}$$

are the creation and annihilation operators. The normalization constants in Eq. (12) come from the classical action integral (entering the functional integral), which is $c^4\kappa^{-1}\int\sqrt{-\det(g)}R$, where $\kappa = 8\pi G$ and G is the Newton constant.

The Hamiltonian is

$$H_R = \int d\mathbf{k}\sum_\zeta c|\mathbf{k}|C(\zeta,\mathbf{k})^+C(\zeta,\mathbf{k})$$

If the gravitational radiation is in equilibrium with light and matter, then it should be described by the Gibbs distribution (the Planck black body law)

$$\hat{\rho}_\beta = Z^{-1}\exp(-\beta H_R)$$

where $\frac{1}{\beta} = KT$, K is the Boltzmann constant and T denotes the temperature.

There may be some deviations from the Planck distribution in cosmological models (see Grishchuk (1989); however, there are also arguments in favor of the Planck distribution of the relic gravitational radiation (Weinberg, 1972, 1988; Parker, 1976). We consider a more general density matrix $\hat{\rho}(H_R)$ as a function of the graviton energy H_R. We introduce a parameter $1/b$ as an energy cutoff. In the Gibbs state we obtain the Planck distribution

$$f_\beta^{\mathrm{PL}}(\hbar c|\mathbf{k}|) \equiv \langle C^+(\mathbf{k})C(\mathbf{k})\rangle_\beta = (\exp(\beta c\hbar|\mathbf{k}|)-1)^{-1} \tag{14}$$

We can see that effectively $1/\beta$ plays the role of the energy cutoff in the Planck distribution. We shall sometimes identify b with β in our discussion.

The correlation functions of α (12) can be computed in the Fock space

$$Tr\big(\alpha_c^a(t,\mathbf{x})\alpha_{c'}^{a'}(0,\mathbf{x}')\hat{\rho}\big) \equiv G_b(\mathbf{x},\mathbf{x}';t)_{cc'}^{aa'}$$
$$= \frac{\hbar\kappa}{2\pi^2c^3}\int d\mathbf{k}\,\frac{1}{|\mathbf{k}|}\delta_{cc'}^{aa'}(k)\cos(\mathbf{k}(\mathbf{x}-\mathbf{x}'))$$
$$\times\left(\left(\frac{1}{2}+f_b(\hbar c|\mathbf{k}|)\right)\cos(c|\mathbf{k}|t) - \frac{i}{2}\sin(c|\mathbf{k}|t)\right) \tag{15}$$

where

$$\delta^{ac}_{bd}(k) = \sum_{\zeta} \overline{\varepsilon^a_b}(k,\zeta)\varepsilon^c_d(k,\zeta) \tag{16}$$

We assume the transverse-traceless gauge for α. This means that we can choose only spatial components of α different from zero. In such a case $\delta(k)$ has the form (see Weinberg (1965) and Rubakov (1982))

$$\delta^{ac}_{bd}(k) = \tilde{\delta}^d_a\tilde{\delta}^b_c + \tilde{\delta}_{ac}\tilde{\delta}^{db} - \tilde{\delta}^b_a\tilde{\delta}^d_c \tag{17}$$

where

$$\tilde{\delta}_{ac}(k) = \delta_{ac} - k_a k_c/\mathbf{k}^2$$

The form (17) of δ is determined by the conditions $k_b\delta^{bd}_{ac}(k) = 0$, $\delta^{ad}_{ac}(k) = 0$, and $\delta^{bc}_{ac}(k) = 0$.

In Eq. (15) f_b is the graviton distribution. Our results for small time and small space separations do not depend essentially on the form of f_b if

$$f_b(k) = \tilde{f}(bk)$$

and if $\tilde{f}$ decays sufficiently fast, for example, $|\tilde{f}(k)| \leq Ak^{-6}$ for a large k. For large time and large space separations the results depend on the singularity of $\tilde{f}(k)$ at $k = 0$. Some singular distributions in inflationary models are discussed in Allen (1988), Allen and Romano (1999), de Garcia Maia (1993), and Sahni (1990).

An expectation value in the vacuum χ is a special case of Eq. (15) corresponding to the limit $b \to \infty$

$$\begin{aligned} G_\infty(\mathbf{x}, t; \mathbf{x}', 0) &\equiv < \chi|\alpha(t,\mathbf{x})\alpha(0,\mathbf{x}')|\chi > \\ &= \kappa c^{-4}\frac{\hbar c}{4\pi^2}\int d\mathbf{k}\,\frac{1}{|\mathbf{k}|}\cos(\mathbf{k}(\mathbf{x}-\mathbf{x}'))\exp(-ic|\mathbf{k}|t) \end{aligned} \tag{18}$$

We can see from Eqs. (15) and (18) that the first term on the r.h.s. of Eq. (15) describes the zero point density (vacuum fluctuations), whereas the second one (f_b) comes from the thermal gravitons in equilibrium with the environment. In general, the vacuum fluctuations cannot be neglected. After a renormalization they contribute to measurable effects. However, we show in the Appendix that renormalized vacuum fluctuations give a negligible contribution to the decoherence. We subtract the vacuum fluctuations in Eq. (15) defining $G_{\mathrm{th}} = G_b - G_\infty$. After this subtraction the correlation function becomes real. We can define a real random field α with the correlation function

$$\left\langle \alpha^a_b(x)\alpha^c_d(x')\right\rangle = G_{\mathrm{th}}(x-x')^{ac}_{bd}$$

where

$$G_{\rm th}(x-x')^{ac}_{bd} = \hbar\kappa c^{-3}(2\pi)^{-2}\int d\mathbf{k}\,|\mathbf{k}|^{-1}\delta^{ac}_{bd}(k)\,\cos(\mathbf{k}(\mathbf{x}-\mathbf{x}'))$$
$$\times\cos((x_0-x_0')|\mathbf{k}|)\,f_b(c|\mathbf{k}|\hbar) \tag{19}$$

The random field α is Gaussian in a linear approximation to gravity.

3. DECOHERENCE

Gravitons interact with all particles. There is no screening of the gravitation force. Hence, the eventual decoherence effect of gravitons will be universal. We define the partial density matrix (averaged over the gravitons)

$$\rho_t(\mathbf{x},\mathbf{x}') = Tr_R(\langle\mathbf{x}|\hat{\rho}(H_R)|\psi_t\rangle\langle\psi_t \mid \mathbf{x}'\rangle) \tag{20}$$

We take ψ in the form (2), where ϕ is a slowly varying function. Hence, in the leading order in $\hbar$ neglecting the dependence of $\phi(x)$ on α

$$\psi(x) \equiv \psi_t(\mathbf{x}) = \exp\left(\frac{i}{\hbar}W\right)\tilde{\phi}(\mathbf{x}-x_0\mathbf{P}/P_0) \tag{21}$$

where $x_0 = ct$ and $\phi(x_0,\mathbf{x}) = \tilde{\phi}(\mathbf{x}-x_0\mathbf{P}/P_0)$ is the solution of the Klein–Gordon equation with the initial condition $\tilde{\phi}$. The average over gravitons takes the form (we skip the tilde over ϕ)

$$\rho_t(\mathbf{x},\mathbf{x}') \equiv \exp(-S(P))\,\exp(iP(x-x')/\hbar)\phi\left(\mathbf{x}-\mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}'-\mathbf{P}\frac{x_0}{P_0}\right)}$$
$$= \exp\big(iP(x-x')/\hbar\big)\,\phi\left(\mathbf{x}-\mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}'-\mathbf{P}\frac{x_0}{P_0}\right)}$$
$$\times\exp\left(-\frac{1}{\hbar^2P_0^2}\int_0^{ct}P^aP_cG^{cd}_{ab}\left(\frac{s-s'}{P_0}\mathbf{P},s-s'\right)P^bP_d\,ds\,ds'\right.$$
$$+\frac{1}{2\hbar^2P_0^2}\int_0^{ct}P^aP_cG^{cd}_{ab}(\mathbf{x}-\mathbf{x}'-(s-s')\mathbf{P}/P_0,s-s')P_dP^b\,ds\,ds'$$
$$\left.+\frac{1}{2\hbar^2P_0^2}\int_0^{ct}P^aP_cG^{cd}_{ab}(\mathbf{x}'-\mathbf{x}-(s-s')\mathbf{P}/P_0,s-s')P_dP^b\,ds\,ds'\right) \tag{22}$$

The form of the Green's function comes from

$$\langle\alpha(\mathbf{q}(\tau,\mathbf{x}),\tau)\,\alpha(\mathbf{q}(s,\mathbf{x}'),s)\rangle = G\left(\mathbf{x}-\mathbf{x}'+\frac{\tau-s}{P_0}\mathbf{P},\tau-s\right) \tag{23}$$

The trace in Eq. (21) can be calculated as an expectation value over the Gaussian random field α (or expressed in the operator formalism by means of the time-ordered products in the Fock space)

$$\langle \exp i\alpha J\rangle = \exp\left(-\frac{1}{2}JGJ\right)$$

where the Green functions G depend on the state under consideration (See Haba and Kleinert, 2001). In particular, in the thermal state with subtracted vacuum fluctuations, $G \rightarrow G_{\mathrm{th}}$. With vacuum fluctuations, $G_{\mathrm{th}} \rightarrow G_{\beta} = G_{\mathrm{th}} + G_{\infty}$ and the time-ordering leads subsequently to the replacement $G_{\infty} \rightarrow G_{\mathrm{F}} = i\Delta_{\mathrm{F}}$ (in the notation of Bjorken and Drell, 1965). The part $\int G_{\mathrm{F}}\, d\mathbf{y}\, d\mathbf{y}$ contains infinities when the paths intersect. After a renormalization the remaining expression gives a negligible contribution to the decoherence (see the Appendix).

If there are N particles then

$$\begin{aligned} W_t(\mathbf{x}_1, \ldots, \mathbf{x}_N) = \sum_{j=1}^{N} \Bigg(& P(j)x(j) + \frac{1}{P_0(j)} P(j)_c P(j)^a \\ & \times \int_0^{ct} \alpha_a^c(\mathbf{x}(j) + s\mathbf{P}(j)P_0(j)^{-1}, ct - s)\, ds \Bigg) \end{aligned}$$

and the density matrix evolves as follows

$$\begin{aligned} \rho_t(\mathbf{x}, \mathbf{x}') \equiv{} & \exp(-S(P)) \exp\left(i \sum_{j=1}^{N} P(j)(x(j) - x'(j))/\hbar\right) \\ & \times \phi\left(\mathbf{x} - \mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}' - \mathbf{P}\frac{x_0}{P_0}\right)} \\ ={} & \exp\left(i \sum_{j=1}^{N} P(j)(x(j) - x'(j))/\hbar\right) \phi\left(\mathbf{x} - \mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}' - \mathbf{P}\frac{x_0}{P_0}\right)} \\ & \times \exp\Bigg(-\frac{1}{2\hbar^2}\int_0^{ct} \sum_{j,k=1}^{N} P(j)^a P(j)_c P_0(j)^{-1} \\ & \times \Bigg(G_{ab}^{cd}\left(\mathbf{x}(j) - \mathbf{x}(k) + \frac{s}{P_0(j)}\mathbf{P}(j) - \frac{s'}{P_0(k)}\mathbf{P}(k), s - s'\right) \\ & + G_{ab}^{cd}\left(\mathbf{x}'(j) - \mathbf{x}'(k) + \frac{s}{P_0(j)}\mathbf{P}(j) - \frac{s'}{P_0(k)}\mathbf{P}(k), s - s'\right) \\ & - G_{ab}^{cd}\left(\mathbf{x}'(j) - \mathbf{x}(k) + \frac{s}{P_0(j)}\mathbf{P}(j) - \frac{s'}{P_0(k)}\mathbf{P}(k), s - s'\right) \end{aligned}$$

$$
\begin{aligned}
&- G_{ab}^{cd}\left(\mathbf{x}(j) - \mathbf{x}'(k) + \frac{s}{P_0(j)}\mathbf{P}(j) - \frac{s'}{P_0(k)}\mathbf{P}(k), s - s'\right)\Bigg)\\
&\times P^b(k) P_d(k) P_0(k)^{-1}\ \ ds\, ds'\Bigg)
\end{aligned}
\tag{24}
$$

where $\mathbf{x} - x_0\mathbf{P}/P_0$ under the argument of ϕ is understood as a vector with components $\mathbf{x}(j) - x_0\mathbf{P}_{(j)}/P_0(j)$.

Let us first consider a single particle and assume that t is small so that we can neglect the integration over s and s' inside the cosine. Let us choose $\mathbf{P}$ as the z-axis and denote the coordinates of $\mathbf{x} - \mathbf{x}' = \mathbf{y} = (\cos\alpha \sin\vartheta, \sin\alpha \sin\vartheta, \cos\vartheta)$. Then, with $\mathbf{k} = |\mathbf{k}|(\cos\phi\ \sin\theta, \sin\phi\ \sin\theta, \cos\theta)$

$$
\begin{aligned}
\mathbf{PP}\Bigg(&\int_0^{ct} G_{\text{th}}((s-s')\mathbf{P}/P_0, s-s')\, ds\, ds'\\
&-\frac{1}{2}\int_0^t G_{\text{th}}(\mathbf{x} - \mathbf{x}' + (s-s')\mathbf{P}/P_0, s-s')\, ds\, ds'\\
&-\frac{1}{2}\int_0^t G_{\text{th}}(\mathbf{x}' - \mathbf{x} + (s-s')\mathbf{P}/P_0, s-s')\, ds\, ds'\Bigg)\,\mathbf{PP}\\
&= t^2\mathbf{PP}(G_{\text{th}}(0,0) - G_{\text{th}}(\mathbf{y},0))\mathbf{PP}\\
&= |\mathbf{P}|^4\hbar\kappa c^{-3}t^2\pi^{-2}\int_0^\infty dk\, k\int_0^1 da\int_0^{2\pi} d\phi\int_0^\pi d\theta \sin\theta\, I(\theta)\\
&\times(1 - \cos(ka|\mathbf{y}|\Theta))\,\tilde{f}(b\hbar ck)
\end{aligned}
\tag{25}
$$

where

$$
\Theta = \cos\phi\ \sin\theta\ \cos\alpha\ \sin\vartheta + \sin\phi\ \sin\theta\ \sin\alpha\ \sin\vartheta + \cos\theta\ \cos\vartheta \tag{26}
$$

and

$$
I(\theta) = |\mathbf{P}|^{-4}(\mathbf{P}^2 - (\mathbf{Pk})^2\mathbf{k}^{-2})^2 = (1 - \cos^2\theta)^2
$$

For small $\mathbf{y}$ the integration over ϕ gives

$$
\int d\phi\, \Theta^2 = \frac{\pi}{2}(\sin^2\vartheta + \cos^2\vartheta\ \cos^2\theta)
$$

Hence, expanding in $\mathbf{y}$ we obtain

$$
\rho_t(\mathbf{x}, \mathbf{x}') = \exp\left(-(A + B\ \sin^2\vartheta)\left(\frac{\hbar}{|\mathbf{P}|}\right)^{-2} l_{dB}^{-2}|\mathbf{x} - \mathbf{x}'|^2(L_{\text{PL}}/l_{dB})^2(\mathbf{P}t/M)^2\right) \tag{27}
$$

where $A > 0$ and $B > 0$ are constants of order 1, $l_{dB} = \hbar cb$ is de Broglie length at temperature T (for $b = \beta$). Then, $\frac{\hbar}{|\mathbf{P}|}$ is particle's wave length at the momentum $\mathbf{P}$, $L_{\mathrm{PL}} = \sqrt{\hbar\kappa/c^3}$ is the Planck length.

For N particles if t is small and the difference in coordinates in Eq. (24) is small then expanding in t and in the coordinates we obtain (assuming $P_0 \simeq Mc$)

$$\begin{aligned}
\rho_t(\mathbf{x}, \mathbf{x}') &\equiv \exp\left(-S(P)\right) \exp\left(i \sum_{j=1}^{N} P(j)(x(j) - x'(j))/\hbar\right) \\
&\quad \times \phi\left(\mathbf{x} - \mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}' - \mathbf{P}\frac{x_0}{P_0}\right)} \\
&= \exp\left(i \sum_{j=1}^{N} (P(j)(x(j) - x'(j))/\hbar)\phi\left(\mathbf{x} - \mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}' - \mathbf{P}\frac{x_0}{P_0}\right)}\right. \\
&\quad \times \exp\left(-\frac{t^2}{2\hbar^2 M^2 c^2} \sum_{j,l=1}^{N} ((\mathbf{x}(j) - \mathbf{x}(l))\gamma(\mathbf{P}(j), \mathbf{P}(l))(\mathbf{x}(j) - \mathbf{x}(l))\right. \\
&\quad + (\mathbf{x}'(j) - \mathbf{x}'(l))\gamma(\mathbf{P}(j), \mathbf{P}(l))(\mathbf{x}'(j) - \mathbf{x}'(l)) \\
&\quad + (\mathbf{x}'(j) - \mathbf{x}(l))\gamma(\mathbf{P}(j), \mathbf{P}(l))(\mathbf{x}'(j) - \mathbf{x}(l)) \\
&\quad \left. + (\mathbf{x}(j) - \mathbf{x}'(l))\gamma(\mathbf{P}(j), \mathbf{P}(l))(\mathbf{x}(j) - \mathbf{x}'(l)))\right)
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
&(\mathbf{x}(j) - \mathbf{x}(l))\gamma(\mathbf{P}(j), \mathbf{P}(l))(\mathbf{x}(j) - \mathbf{x}(l)) \\
&\quad = \int d\mathbf{k}\, |\mathbf{k}|^{-1} f_b(c\hbar k)(\mathbf{k}(\mathbf{x}(j) - \mathbf{x}(l)))^2 \mathbf{P}(j)\mathbf{P}(l)\delta(k)\mathbf{P}(j)\mathbf{P}(l)
\end{aligned}$$

The formula simplifies if all the momenta are equal (and nonrelativistic, i.e., $P_0 \simeq Mc$) then

$$\begin{aligned}
\rho_t(\mathbf{x}, \mathbf{x}') &= \exp\left(iP \sum_{j=1}^{N} (x(j) - x'(j))/\hbar\right) \phi\left(\mathbf{x} - \mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}' - \mathbf{P}\frac{x_0}{P_0}\right)} \\
&\quad \times \exp\left(-\frac{t^2}{2\hbar^2 M^2 c^2} |\mathbf{P}|^4 \sum_{j,k=1}^{N} \left((A + B\,\sin^2 \vartheta_{jk})|\mathbf{x}(j) - \mathbf{x}(k)|^2\right.\right. \\
&\quad + (A + B\,\sin^2 \vartheta''_{jk})|\mathbf{x}'(j) - \mathbf{x}'(k)|^2 + (A + B\,\sin^2 \vartheta'_{jk})|\mathbf{x}'(j) - \mathbf{x}(k)|^2 \\
&\quad \left.\left. + (A + B\,\sin^2 \vartheta'_{kj})|\mathbf{x}(j) - \mathbf{x}'(k)|^2\right)\right)
\end{aligned} \tag{29}$$

where ϑ_{jk} is the angle between $\mathbf{P}$ and $\mathbf{x}(j) - \mathbf{x}(k)$, ϑ''_{jk} is the angle between $\mathbf{P}$ and $\mathbf{x}'(j) - \mathbf{x}'(k)$, and ϑ'_{jk} is the angle between $\mathbf{P}$ and $\mathbf{x}(j) - \mathbf{x}'(k)$.

To obtain a simple bound on the density matrix (29), let us assume additionally that the particles have approximately the same position $\mathbf{x}(j) = \mathbf{x}(k) = \mathbf{x}$ and $\mathbf{x}'(j) = \mathbf{x}'(k) = \mathbf{x}'$. One can achieve this by choosing in the formula

$$\langle A\rangle = (Tr\rho)^{-1}Tr(\rho A)$$

the observable A, which is of the form $|\chi><\chi|$ with

$$\chi(\mathbf{x}(1), \ldots, \mathbf{x}(N)) = \exp\left(i\mathbf{P}\sum \mathbf{x}(j)/\hbar\right)\prod_{j=1}^{N-1}\exp\left(-\frac{1}{2\epsilon}(\mathbf{x}(j) - \mathbf{x}(N))^2\right)$$

Then, we obtain the bound

$$|\rho_t(\mathbf{x}, \mathbf{x}')| \leq \left|\phi\left(\mathbf{x} - \mathbf{P}\frac{x_0}{P_0}\right)\overline{\phi\left(\mathbf{x}' - \mathbf{P}\frac{x_0}{P_0}\right)}\right|$$
$$\exp\left(-N\frac{t^2}{2\hbar^2M^2c^2}|\mathbf{P}|^4(A + B\ \sin^2\vartheta')|\mathbf{x} - \mathbf{x}'|^2\right)$$

For a large time and large $|\mathbf{x} - \mathbf{x}'|$ we can obtain explicit formulas for G_{th} if $\mathbf{y} = \mathbf{x} - \mathbf{x}' \| \mathbf{P}$. We apply Eq. (22) and the formula

$$A^{-1}\sin A = \int_0^1 da\ \cos(aA)$$

We perform the integral over time first. Then, for nonrelativistic momenta

$$\begin{aligned} S(P) = 2|\mathbf{P}|^4\frac{1}{M^2\hbar}\frac{\kappa}{c^5\pi}\int_0^1 da\int_0^\infty\frac{dk}{k}\int_0^\pi d\theta\ \sin\theta\, I(\theta)\, f_b(\hbar ck) \\ \times (2(1 - \cos(tck)) - 2\cos(ka\ \cos\theta|\mathbf{y}|) \\ + \cos(ka\ \cos\theta|\mathbf{y}| + ckt) + \cos(-ka\ \cos\theta|\mathbf{y}| + ckt)) \end{aligned} \tag{30}$$

For $ct \gg |\mathbf{y}|$ we can neglect the terms with t under the cosines in Eq. (30). There remains

$$\begin{aligned} S = \frac{4}{M^2\hbar}\frac{\kappa}{c^5\pi}|\mathbf{P}|^4\int_0^1 da\int_0^\infty\frac{dk}{k}\int_0^\pi d\theta \\ \times \sin\theta\, I(\theta)\,\tilde{f}(b\hbar ck)(1 - \cos(ka\ \cos\theta|\mathbf{y}|)). \end{aligned} \tag{31}$$

If $|\mathbf{y}| \ll l_{dB}$ is small and $|\mathbf{P}|$ is small in comparison to Mc (and t is large) then the integral (31) gives

$$\rho_t(\mathbf{x}, \mathbf{x}') \approx \exp\left(-AM^{-2}c^{-2}\hbar^{-2}|\mathbf{P}|^4L_{\text{PL}}^2 l_{dB}^{-2}|\mathbf{x} - \mathbf{x}'|^2\right) \tag{32}$$

with a certain constant A of order 1.

For a small time t and a large $|\mathbf{y}|$ we omit the terms with $\mathbf{y}$ in the integral (30). There remains

$$S(P) = 4|\mathbf{P}|^4 \frac{1}{M^2\hbar}\frac{\kappa}{c^5\pi}\int_0^1 da \int_0^\infty \frac{dk}{k}\int_0^\pi d\theta \sin\theta I(\theta)\, f_b(\hbar ck)(1-\cos(tck))$$

Hence, for $|\mathbf{y}| \gg l_{dB} \gg ct$ we obtain

$$\rho_t \approx \exp\left(-A(L_{\mathrm{PL}}/l_{dB})^2\left(\frac{\hbar}{|\mathbf{P}|}\right)^{-2}(\mathbf{P}t/M)^2\right) \tag{33}$$

Let us consider the large $|\mathbf{y}|$ in Eq. (30) assuming that t is also large. Now, the behaviour of $S(\mathbf{P})$ for a large time depends essentially on the energy distribution. We restrict the discussion to the Planck distribution. We apply the formula

$$1-\cos w = w\int_0^1 d\gamma \sin(\gamma w)$$

and the formula 3.911 of Gradstein and Ryzhik (1971)

$$\int_0^\infty du \sin(au)(\exp(\beta u)-1)^{-1} = \frac{\pi}{2\beta}\coth\left(\frac{\pi a}{\beta}\right) - \frac{1}{2a}$$

For $|\mathbf{y}| \gg ct \gg l_{dB}$ we neglect the oscillatory terms depending on $\mathbf{y}$. Then, we obtain

$$\begin{aligned} S(P) &= 2\frac{ct}{M^2\hbar^2}|\mathbf{P}|^4\frac{\kappa}{c^4}\hbar/\pi\int_0^1 d\gamma\int_0^\infty dk\int_0^\pi d\theta\sin\theta I(\theta) \\ &\quad\times \sin(tck\gamma)(\exp(\beta\hbar ck)-1)^{-1} \\ &= \frac{\kappa}{M^2c^5\hbar^2}|\mathbf{P}|^4\hbar/\pi\int_0^t d\gamma\left(\frac{\pi}{\beta\hbar}\coth\left(\frac{\pi\gamma}{\beta\hbar}\right)-\frac{1}{\gamma}\right) \\ &= \frac{\kappa}{M^2c^5\hbar^2}|\mathbf{P}|^4\hbar/\pi\ln\left(\frac{\beta\hbar}{\pi t}\sinh\left(\frac{\pi t}{\beta\hbar}\right)\right) \\ &\approx \kappa\frac{|\mathbf{P}|^4}{M^2c^5}\hbar^{-3}\frac{ct}{l_{dB}} = L_{\mathrm{PL}}^2\frac{|\mathbf{P}|^2}{M^2c^3}(\hbar|\mathbf{P}|^{-1})^{-2}\frac{ct}{l_{dB}} \end{aligned} \tag{34}$$

For $ct \gg |\mathbf{y}| \gg l_{dB}$ and intermediate $|\mathbf{y}|$ we neglect the terms with t under the difference of cosines in Eq. (30). There remains

$$\begin{aligned} S(P) &= 2c^{-5}\frac{\kappa}{M^2\hbar^2}|\mathbf{P}|^4\hbar/\pi\int_0^1 da\int_0^\infty\frac{dk}{k}\int_0^\pi d\theta\sin\theta I(\theta) \\ &\quad\times(\exp(\beta\hbar ck)-1)^{-1}(1-\cos(ka\,\cos\theta|\mathbf{y}|)) \end{aligned} \tag{35}$$

Then, the k-integral gives

$$S = \frac{\kappa}{M^2c^5\hbar^2}|\mathbf{P}|^4\hbar/c\pi \int_0^1 da\left(\int_0^1 d\gamma \int_0^\pi d\cos\theta I(\theta)\right.$$
$$\left.\times\left(\frac{\pi a\ \cos\theta|\mathbf{y}|}{\beta\hbar c}\coth\left(\frac{\pi a\ \cos\theta|\mathbf{y}|\gamma}{\beta\hbar c}\right)-\frac{1}{\gamma}\right)\right)$$
$$\approx 2\hbar^{-1}\kappa\frac{|\mathbf{P}|^4}{2M^2c^5}\int_0^{\frac{\pi}{2}} d\cos\theta I(\theta)\cos\theta\frac{|\mathbf{y}|}{l_{dB}}$$
$$= \frac{|\mathbf{P}|^2}{12M^2c^2}L_{\mathrm{PL}}^2(\hbar|\mathbf{P}|^{-1})^{-2}\frac{|\mathbf{y}|}{l_{dB}} \tag{36}$$

for a large $|\mathbf{y}|$ such that $ct \gg |\mathbf{y}| \gg l_{dB}$.

If (as in Eq. (34))

$$\rho_t(\mathbf{x},\mathbf{x}') \simeq \exp(i\mathbf{P}(\mathbf{x}-\mathbf{x}')/\hbar) - at|\mathbf{P}|^4)$$

then

$$\partial_t\rho = -a[\mathbf{P}^2,[\mathbf{P}^2,\rho]]$$

with

$$a = \frac{c}{l_{dB}}\frac{L_{\mathrm{PL}}^2}{\hbar^2M^2c^2}$$

This could be a version of a primary state diffusion equation (Diosi, 1989; Milburn, 1991; Percival and Struntz, 1997) causing decoherence and the wave function reduction. In our interpretation the diffusion results from the gravitational background (for a detailed study of this equation see Haba and Kleinert, 2001).

Note that the formula (34) comes out from the approximation

$$G_{\mathrm{th}}(\mathbf{x},\mathbf{x}',s-s') = \frac{cL_{\mathrm{PL}}^2}{l_{dB}}\delta(s-s')$$

which could have been justified by an approximation $f_{\mathrm{PL}} \approx l_{dB}^{-1}$. If α is approximated by the white noise then Eq. (1), the Klein–Gordon equation, takes the form of a random Schrödinger equation (with the Ansatz $\psi = \exp(iPx/\hbar)\phi$)

$$\partial_t\phi = -\frac{i}{2\hbar M}\mathbf{P}^2\phi - L_{\mathrm{PL}}\frac{i}{\hbar M}\alpha^{jl}P_jP_l\phi$$

where $\mathbf{P} = -i\hbar\nabla$.

Note that without the assumption $f \sim 1/k$ for small k (true for the Planck distribution) we would not obtain Eq. (34) but rather t^γ. Then, there would be no diffusion equation.

For real systems there are stronger sources of the decoherence than the gravitational one, for example, a scattering on relic photons, sun photons, and air molecules. However, in principle we can screen a system against these disturbances. The gravitons effect is universal, with no possibility of screening. It is small for elementary particles but as we have shown, the decoherence rate increases at least as N for a cluster of N particles so it becomes efficient for macroscopic bodies. We suggest that the gravitational decoherence could be used as a universal mechanism explaining the emergence of the classical world from the quantum microphysics.

4. THE DISAPPEARANCE OF THE ENTANGLEMENT

We consider the Einstein–Podolsky–Rosen wave function of two particles (Einstein *et al.* (1935) and Cohen (1997))

$$\psi(\mathbf{x}(1), \mathbf{x}(2)) = \exp\left(\frac{i}{\hbar}\mathbf{P}(1)\mathbf{x}(1) + \frac{i}{\hbar}\mathbf{P}(2)\mathbf{x}(2) - (\mathbf{x}(1) - \mathbf{x}(2) - \mathbf{a})^2/2\epsilon\right) \tag{37}$$

This function approximates EPR δ-function when $\epsilon \to 0$. In the momentum space

$$\begin{aligned}\tilde{\psi}(\mathbf{k}(1), \mathbf{k}(2)) &= \delta(\mathbf{P}(1) + \mathbf{P}(2) - \mathbf{k}(1) - \mathbf{k}(2)) \\ &\quad \times \exp\left(-\frac{\epsilon}{2\hbar^2}(\mathbf{k}(1) - \mathbf{P}(1))^2 + \frac{i}{\hbar}\mathbf{a}(\mathbf{P}(1) - \mathbf{k}(1))\right)\end{aligned} \tag{38}$$

Hence, a measurement of $\mathbf{k}(1)$ determines $\mathbf{k}(2) = -\mathbf{k}(1) + \mathbf{P}(1) + \mathbf{P}(2)$.

We prepare the state (37) at $t = 0$ and calculate its time evolution in the environment of gravitons (that remain unobserved). As a result we obtain the mixed state ρ_t. For an interpretation of this mixed state it is useful to define the Wigner function (Wigner, 1932)

$$\begin{aligned}&\mathcal{W}_t(\mathbf{k}(1), \mathbf{k}(2); \mathbf{q}(1), \mathbf{q}(2)) \\ &\quad = (2\pi\hbar)^{-6}\int d\mathbf{y}(1)\, d\mathbf{y}(2) \rho_t(\mathbf{q} + \mathbf{y}/2, \mathbf{q} - \mathbf{y}/2) \exp(i\mathbf{y}(1)\mathbf{k}(1)/\hbar + i\mathbf{y}(2)\mathbf{k}(2)/\hbar)\end{aligned} \tag{39}$$

At $t = 0$ we have

$$\begin{aligned}&\mathcal{W}(\mathbf{k}(1), \mathbf{k}(2); \mathbf{q}(1), \mathbf{q}(2)) \\ &\quad = (2\pi\epsilon)^{-3/2}\exp\left(-\frac{1}{\epsilon}(\mathbf{q}(1) - \mathbf{q}(2) - \mathbf{a})^2\right) \times \delta(\mathbf{P}(1) + \mathbf{P}(2) - \mathbf{k}(1) - \mathbf{k}(2))\end{aligned}$$

If we insert the result (29) (for two particles) into Eq. (39)

$$\rho_t \approx \exp\left(-\frac{t^2}{2\Gamma}(\mathbf{y}(1)^2 + \mathbf{y}(2)^2)\right)$$

with a (momentum-dependent) Γ determined by Eq. (29), then we obtain

$$\begin{aligned}
&\mathcal{W}_t(\mathbf{k}(1), \mathbf{k}(2); \mathbf{q}(1), \mathbf{q}(2)) \\
&\quad = (2\pi\epsilon)^{-3/2} \exp\left(-\frac{1}{\epsilon}\left(\mathbf{q}(1) - \mathbf{q}(2) - (\mathbf{P}(1) - \mathbf{P}(2))\frac{t}{M} - \mathbf{a}\right)^2\right)(2\pi t^2/\Gamma)^{-3/2} \\
&\qquad \times \exp\left(-\frac{\Gamma}{4t^2}(\mathbf{P}(1) + \mathbf{P}(2) - \mathbf{k}(1) - \mathbf{k}(2))^2\right) \exp\left(-\frac{1}{4}\left(\frac{t^2}{\Gamma} + \frac{1}{\epsilon}\right)^{-1}\right. \\
&\qquad \left. \times (\mathbf{P}(1) - \mathbf{P}(2) - \mathbf{k}(1) + \mathbf{k}(2))^2\right) \qquad (40)
\end{aligned}$$

The result (40) means that the quantum coherence is lost in time $\approx \sqrt{\Gamma}$ after the state preparation. From Eq. (29) it follows that $\Gamma \approx 1/\sqrt{N}$; hence, the decoherence time can be short for a large N. The measurement of $\mathbf{k}(1)$ allows to determine $\mathbf{k}(2)$ with an unavoidable error that grows with time (hence also with the space separation of particles). The error has a nonzero lower bound and a finite upper bound, because according to Eq. (34) the decay of $\rho_t(\mathbf{x}, \mathbf{x}')$ as $|\mathbf{x} - \mathbf{x}'| \to \infty$ is time independent.

APPENDIX

When the operator formalism is applied, then the formula (22) results from a representation of the density matrix expectation values by the expectation values of the time-ordered products of quantum fields in the Fock space. We use the following conventions of Bjorken and Drell (1965) (T denotes the time-ordered product)

$$\langle 0|T(\alpha(x')\alpha(x))|0\rangle = i\Delta_F(x' - x)$$

$$\langle 0|T(\exp(iJ\alpha))|0\rangle = \exp\left(-\frac{i}{2}J\Delta_F J\right)$$

Then, in our notation

$$G_F(x' - x) = i\Delta_F(x' - x)$$

In terms of Fourier integrals

$$\Delta_F(x' - x) = -i\frac{1}{2}(2\pi)^{-3}\int d\mathbf{k}\,|\mathbf{k}|^{-1}\cos(\mathbf{k}(\mathbf{x}' - \mathbf{x}))\exp(-ic|\mathbf{k}||t' - t|)$$

We perform calculations with an ultraviolet cutoff Λ. Then, we obtain (for simplicity of the formulas we make an approximation $P_0/Mc \simeq 1$)

$$\begin{aligned}
S_F = 2\frac{1}{M^2\hbar^2}\mathbf{P}\mathbf{P}\bigg(&\int_0^t G_F((s-\tau)\mathbf{P}/M, s-\tau)\,ds\,d\tau \\
&- \int_0^t G_F(\mathbf{x} - \mathbf{x}' + (s-\tau)\mathbf{P}/M, s-\tau)\,ds\,d\tau
\end{aligned}$$

$$
\begin{aligned}
& - \int_0^t G_{\mathrm{F}}(\mathbf{x}' - \mathbf{x} + (s-\tau)\mathbf{P}/M, s-\tau)\, ds\, d\tau\Big)\Big)\mathbf{P}\mathbf{P} \\
&= \hbar c \frac{1}{2}(2\pi)^{-3}(-i)ct \int d\mathbf{k}\, (\mathbf{P}^2 - (\mathbf{P}\mathbf{k})^2\mathbf{k}^{-2})^2 \\
&\quad \times (c^2|\mathbf{k}|^2 - (\mathbf{P}\mathbf{k})^2/M^2)^{-1}(1 - \cos(\mathbf{k}(\mathbf{x}' - \mathbf{x}))) \\
&\quad + \hbar c \frac{1}{2}(2\pi)^{-3} \int d\mathbf{k}\, (\mathbf{P}^2 - (\mathbf{P}\mathbf{k})^2\mathbf{k}^{-2})^2 |\mathbf{k}|^{-1}((c|\mathbf{k}| + \mathbf{P}\mathbf{k}/M)^{-2} \\
&\quad \times (1 - \exp(-it(c|\mathbf{k}| + \mathbf{P}\mathbf{k}/M))) \\
&\quad + (c|\mathbf{k}| - \mathbf{P}\mathbf{k}/M)^{-2}(1 - \exp it(c|\mathbf{k}| - \mathbf{P}\mathbf{k}/M)))(1 - \cos(\mathbf{k}(\mathbf{x}' - \mathbf{x}))) \qquad (41)
\end{aligned}
$$

With an ultraviolet cutoff $|\mathbf{k}| \leq \Lambda$ on the wave number, $|\exp(-S_{\mathrm{F}}(P))|$ behaves as follows: If t is large and $|\mathbf{x} - \mathbf{x}'|$ small ($\Lambda|\mathbf{x} - \mathbf{x}'| \leq 1/2$) then

$$|\exp(-S_{\mathrm{F}}(P))| \approx \exp(-a|\mathbf{x} - \mathbf{x}'|^2)$$

If $|\mathbf{x} - \mathbf{x}'|$ is large and t small ($|ct|\Lambda \leq 1/2$) then

$$|\exp(-S_{\mathrm{F}}(P))| \approx \exp(-a|t|^2)$$

If both t and $|\mathbf{x} - \mathbf{x}'|$ are small then

$$|\exp(-S_{\mathrm{F}}(P))| \approx \exp(-a^2|t|^2|\mathbf{x} - \mathbf{x}'|^2)$$

However, in these formulas $a \approx \Lambda^2$. Hence, such a behaviour would be valid only for a very small t or $|\mathbf{x} - \mathbf{x}'|$. In fact, the cutoff sets a length scale Λ^{-1} within which the variation of time and space coordinates should be considered. Beyond this scale the real variation of S_{F} is logarithmic. We can see this from the integral entering the formula (39)

$$
\begin{aligned}
\int_0^{\Lambda} \frac{dk}{k}(1 - \cos ckt) &= \int_0^1 \frac{dk}{k}(1 - \cos k) + \int_1^{ct\Lambda} \frac{dk}{k}(1 - \cos k) \\
&= \mathrm{const} + \ln(ct\Lambda) \qquad (42)
\end{aligned}
$$

Hence, the variation of S_{F} is slow in comparison to the black body part S (that behaves in the same way but for larger time and space intervals). It follows that the termal part G_{th} of the Green's function determines the leading behaviour of the density matrix.

If the ultraviolet cutoff is removed and an infinite constant independent of t and $\mathbf{x}$ is subtracted then the remaining part of $\exp(-S)$ varies slowly (logarithmically) for larger t and $|\mathbf{x} - \mathbf{x}'|$.

It is sometimes suggested that quantum gravity sets an ultraviolet cutoff $k \leq 1/L_{\mathrm{PL}}$, that is, $\Lambda = 1/L_{\mathrm{PL}}$. In such a case the part corresponding to the zero-temperature Green's function G_{F} determines the behaviour of the density matrix only for $|\mathbf{x} - \mathbf{x}'| \leq L_{\mathrm{PL}}$ and $c|t| \leq L_{\mathrm{PL}}$. At finite temperature the thermal Green's function behaves in the same way for a larger space and time intervals determined by l_{dB}, which becomes the relevant length scale for decoherence.

REFERENCES

Allen, B. (1988). *Physical Review D* **37**, 2078.
Allen, B. and Romano, J. D. (1999). *Physical Review D* **59**, 102001.
Anastopoulos, C. (1996). *Physical Review D* **54**, 1600.
Bjorken, J. D. and Drell, S. D. (1965). *Relativistic Quantum Fields*, McGraw-Hill, New York.
Cohen, O. (1997). *Physical Review A* **56**, 3484.
de Garcia Maia, M. R. (1993). *Physical Review D* **48**, 647.
Diosi, L. (1989). *Physical Review A* **40**, 1165.
Einstein, A., Podolsky, B., and Rosen, N. (1935). *Physical Review* **47**, 777.
Ellis, J., Mohanty, S., and Nanopoulos, D. V. (1989). *Physics Letters B* **221**, 113.
Gradshtein, I. S. and Ryzhik, I. M. (1971). *Tables of Integrals, Sums, Series and Products*, Nauka, Moscow, (in Russian).
Grishchuk, L. P. (1989). Uspekhi Fizitseskih Nauka **156**, 297 (in Russian), [Soviet Physics Uspekhi **31**, 940 (1989)].
Haba, Z. and Kleinert, H. (2001). Quantum Liouville and Langerin Equations for Gravitational Radiation Damping, preprint quant-ph/0101006.
Milburn, G. (1991). *Physical Review A* **44**, 5401.
Parker, L. (1976). Nature **261**, 20.
Percival, I. C. and Struntz, W. T. (1997). Proceedings of The Royal Society *A* **453**, 431.
Rubakov, V. A., Sazhin, M. V., and Veryaskin, A. V. (1982). *Physics Letters* **115B**, 189.
Sahni, V. (1990). *Physical Review D* **42**, 453.
Stern, A., Aharonov, Y., and Imry, Y. (1990). *Physical Review A* **41**, 3436.
Weinberg, S. (1965). *Physical Review* **138**, B988.
Weinberg, S. (1972). *Gravitation and Cosmology*, Wiley, New York.
Weinberg, S. (1988). *The First Three Minutes*, Basic Books, New York.
Wigner, E. (1932). *Physical Review* **40**, 749.
Zurek, W. H. (1982). *Physical Review D* **26**, 1862.